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Correlation of noisy images

James A. Shine

Eugene A. Margerum

JUNE 1980

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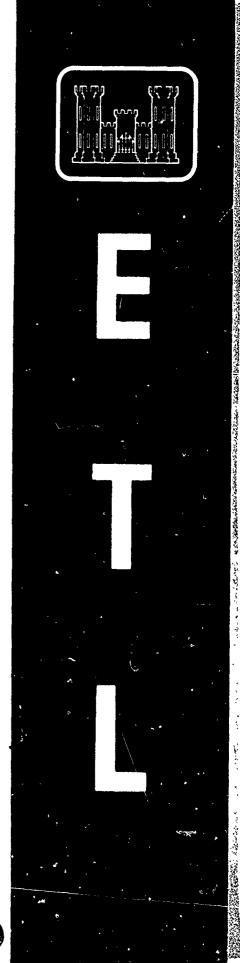
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Filtering

Fourier Transform

Image Processing

ABSTRACT (Continue on reverse side if necessary and identify by block number) A computer program can simulate star-shaped images and a correlation between two different images on 128 by 128 matrices. The Fourier transform, in a timesaving algorithm, is used to carry out the correlation. Different factors in the image (contrast, noise, size) are varied and the effects are observed, The results showed good correlation except when the contrast was low and the noise was high. Using a filtering function in the correlation process produced good results in improving poor correlation patterns.

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PREFACE

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The work reported on was done under DA Project 4A161102B52C, Task S3, Work Unit 0009, under the supervision of Dr. Frederick W. Rohde, Team Leader, Center for Theoretical and Applied Physical Sciences; and Mr. Melvin Crowell, Jr., Director, Research Institute.

COL Daniel L. Lycan, CE was the Commander and Director and Mr. Robert P. Macchia was Technical Director of the Engineer Topographic Laboratories during the study and report preparation.

The authors are grateful to Robert Pazak, Robert Matos, and Barbara Brooke for their assistance in reviewing the computer programs and to Michael McDonnell for his helpful comments on the report.

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CORRELATION OF NOISY IMAGES

INTRODUCTION

Purpose • The purpose of this study was to simulate star-shaped patterns and to observe whether any effects on correlation occurred between an "ideal" pattern and a "real" pattern when different factors affecting the pattern were varied.

Background • Real-world guidance systems (i.e. self-guided missiles) often determine their course of motion by taking continuous images of their surroundings, correlating these images with a "target" image contained within the system, and altering the course of motion (if necessary) towards the region that produces the strongest correlation, or correlation peak. The system's target image is constant and noise-free, but the incoming images often contain extraneous signals (noise) that can disrupt the correlation pattern and produce false peaks.

A correlation process was simulated by forming two matrices representing two separate images. One matrix had random noise scattered throughout, and the other matrix had no such noise; otherwise, the matrices were identical. Four different factors were then tested for their effects on correlation:

- 1. Amount of contrast between the pattern and its background.
- 2. Amount of random noise added to pattern and background.
- 3. Relative size of the pattern.
- 4 Application of a "filtering" function (of variable strength) to the pattern and background during the correlation process.

The correlation was expected to be good except where there was very little contrast between the pattern and the background and a lot of noise. It was also expected that the filtering function would help improve correlation in those cases when it was poor.

THEORY

Photographic images can be simulated on computers by using a two-dimensional matrix, or array, in which each position is given by a numerical value proportional to the lightness or darkness of the corresponding small square area of the photograph. This program simulates star-shaped figures by first mapping out a boundary around a central region, and then assigning greater values to inner positions than to those outside. The boundary was measured from the center of the array, and the center-to-boundary distance was computed as

$$L = R + A \cos(Nx) \tag{1}$$

where R(radius), A(arm length), and N(number of arms) were input into the program, and x was the angle between a line out to some boundary point and a horizontal reference line. Both the inner and outer values were subject to random variations ranging up to a certain value that was also input into the program, thus simulating photographic "noise." These variations could be either positive or negative.

To simulate image correlations, two separate arrays of the same pattern were generated, one with noise and the other with no noise. (This is analogous to comparing a photograph of an object with a perfect image of that object and seeing how well they match up.) A correlation array h can be computed directly from the two image arrays, f and g, by the formula

$$h_{jk} = (1/N) \sum_{n=1}^{N} \sum_{m=1}^{N} f_{mn} g_{m-j, n-k}$$
 (2)

(Note: f, g, and h are all assumed periodic with period N, so $f_{m,n} = f_{m+N,n} = f_{m,n+N}$ and similarly for g and h).

To save computer time, a similar transformation was used, the convolution

$$h_{jk} = \frac{1}{N} \sum_{n=1}^{N} \sum_{m=1}^{N} f_{mn} g_{j-m,k-n} = \frac{1}{N} \sum_{r=1}^{N} \sum_{m=1}^{N} f_{mn} \hat{g}_{m-j,n-k}$$
 (3)

 $(g_{a,b} = g_{-a,-b})$. Since the arrays are two-dimensional, rotating one array 180° and taking a convolution is the same as taking a correlation. The convolution has the useful property that its discrete Fourier transform

$$H_{jk} = \frac{1}{N} \sum_{b=1}^{N} \sum_{a=1}^{N} e^{-\frac{2\pi i(a_{j-bk})}{N}} h_{ab}$$
 (4)

is a simple product of the Fourier transforms of the two image arrays,

$$H_{jk} = \frac{1}{N} F_{jk} G_{jk}$$
 (5)

By using the Fast Fourier Transform (FFT), the Fourier transforms of any of the arrays can be computed in substantially fewer steps than a direct correlation, as in equation 2. Because of the resulting savings of computer time, the correlation was accomplished by the following steps:

- 1. Flip one of the image arrays.
- 2. Take the Fourier transforms of both arrays using the FFT.
- 3. Multiply the transformed arrays together as in equation 5 to produce the correlation/convolution transform matrix H.
- 4. Take the inverse Fourier transform of H to produce the correlation matrix h.

The values in h were then normalized on a scale of 0 to 10 and truncated, their integer parts being printed out with odd numbers left blank for better contrast.

Some of the noisy correlations were subjected to a filter function, the sharpness of which was dependent on an input value W. If an array size of 128 x 128 is used, then W can characterize the width of the filtering function

$$S_{\ell} = \frac{1}{1 - e^{-\alpha^2}} \left[e^{-(\frac{\ell-1}{\sigma})^2} + e^{-(\frac{129 - \ell}{\sigma})^2} \right]; \quad 1 \le \ell \le 128$$
 (6)

where
$$\sigma = 128 \text{ W/E}$$
 and $a = 128/\sigma = \text{E/W}$

with E being the length of an edge of the array. (A small value of W produced a very sharp filter, and a large value resulted in a wider, less effective one). If the Fourier transform of a matrix being correlated is given by F_{jk} , the corresponding filter function is obtained by

$$\widetilde{F}_{jk} = S_j S_k F_{jk}; \qquad 1 \leq j, k \leq 128$$
 (7)

where the filtering has been performed over the first period. The filtering function will tend to suppress the higher frequency components of the image, while leaving the lower frequency components alone. Since the noise is largely in the high frequency range, it will be greatly suppressed relative to the essential image information contained in the low frequency range.

If considered to be periodic, the filtering function is symmetrical with respect to the origin. This was the primary reason for using this function, which is a combination of two Gaussian functions (one at each end of the period). A single Gaussian centered at the origin would not be symmetrical over the period, and one placed at the middle of the period (j = 64 or j = 65) would introduce phase terms into the correlation results.

The problem of aliasing (false correlation signals resulting from overlap of images with one another while performing the correlation) was considered in this work, but the signal level in the overlap region was so low that it was insignificant in comparison to the correlation values; thus it could be neglected.

RESULTS

When correlation was done with a noiseless image, the correlation pattern was very regular, with a sharp central peak fading quickly into a uniform background. This was the case whether the contrast between the star and its background was quite noticeable (as in figure 1), somewhat noticeable (as in figure 2), or only slightly noticeable (as in figure 3). Adding noise to a high-contrast figure broke up the edges of the correlation pattern, but the central peak was still quite sharp and visible. Noise in a low-contrast figure disrupted the entire pattern, although there was still a slight indication of a central peak area (see figures 4 and 5).

Large amounts of noise are needed to cause this pattern disruption; smaller amounts do proportionally less damage to the correlation pattern. The progression of the pattern degradation can be seen for both high-contrast and low-contrast figures (see figures 6-13).

The smallness of the star also affected pattern disruption; the larger stars maintained their central peak much better than the smaller ones under the deluge of heavy noise (compare figures 5, 13, and 14).

Finally, our filter function did an excellent job of restoring badly disrupted patterns. The "sharpness" of the filter could be varied; a very sharp filter (small value of W) restored the worst patterns to smoothness; however, wider filters (large values of W) did not work as well.

Table 1 contains a complete listing of the numerical values used in the programs to produce each figure. Figure 18 contains a sample printout of one of the stars.

TABLE 1. Values for Correlation Figures

Figure	R	A	VIN,VIN2	VOUT, VOUT2	RNIN2	RNOUT2	W(where used)
l	20	10	8	2	0	0	
2	20	10	6	5	0	0	
3	20	10	6	5.5	0	0	****
4	20	10	8	2	5	5	****
5	20	10	6	5.5	5	5	
6	10	5	8	2	0	0	****
7	10	5	8	2	1	1	w
8	10	5	8	2	3	3	
9	10	5	8	2	5	5	
10	10	5	6	5.5	0	0	••••
11	10	5	6	5.5	1	1	****
12	10	5	6	5.5	2	2	-
13	10	5	6	5.5	5	5	
14	7	3.5	6	5.5	5	5	****
15	7	3.5	6	5.5	5	5	3
16	7	3.5	6	5.5	5	5	10
17	7	3.5	6	5.5	5	5	100

RNIN = RNOUT = 0 for all 17 figures; NARMS = 6 for all 17 figures

CONCLUSIONS

- 1. The simulation produced good, centralized correlation patterns in cases where little or no noise was involved.
- 2. Correlation became poor only when contrast between the figure and its background was low (less than 10 percent), and the noise levels were high (greater than 90 percent of peak values).
- 3. The filtering function did an excellent job in restoring poor correlations, which should make it useful for future applications.

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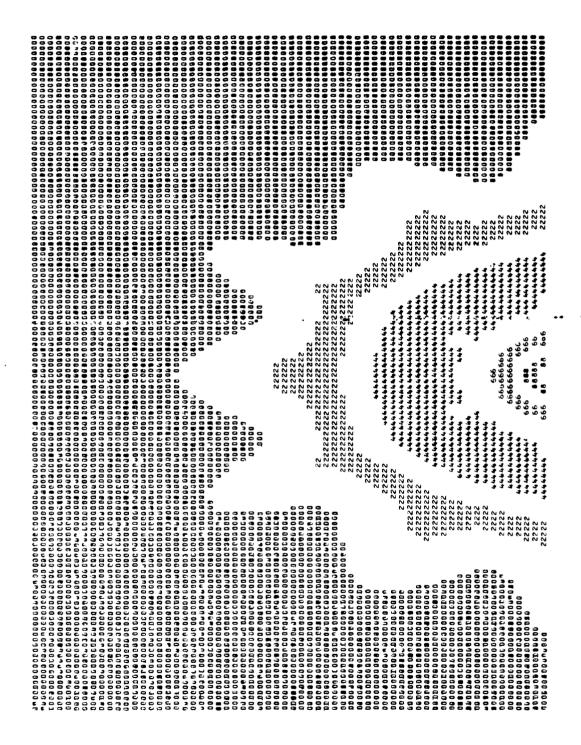


FIGURE 1. R-20, A-10, VIN.VIN2-8, VOUT.VOUT2-2, RNIN2-0, RNOUT2-0, W-N/A.

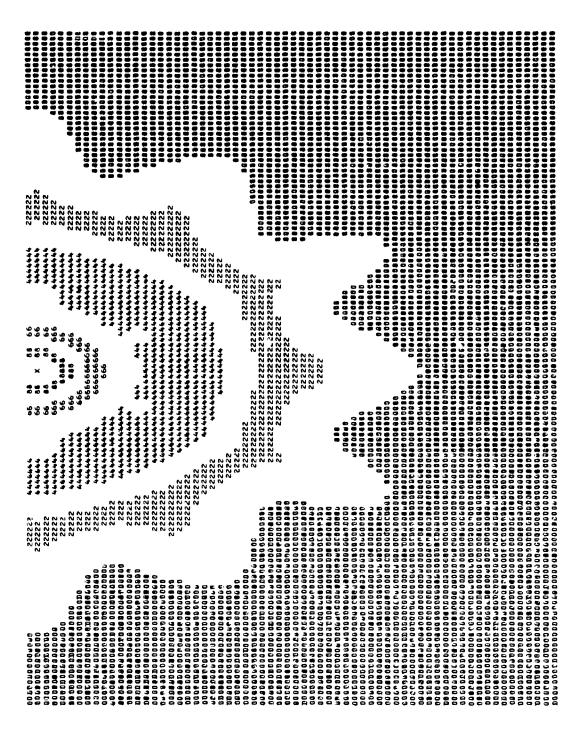


FIGURE 1. Continued

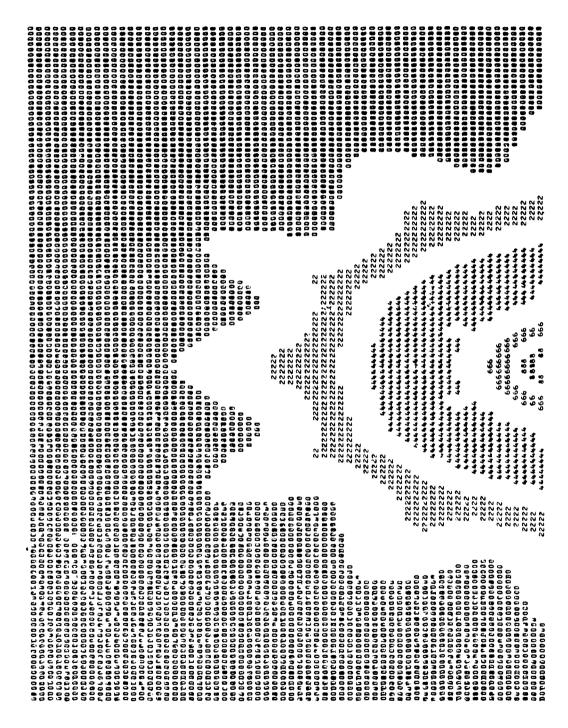


FIGURE 2. R--20, A--10, VIN,VIN2-6, VOUT,VOUT2--5, RNIN2-0, RNOUT2-0, W--N/A.

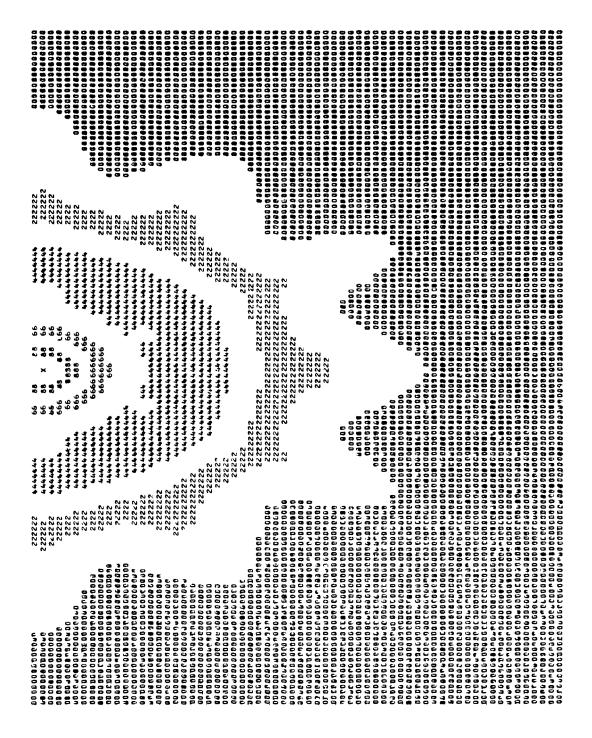


FIGURE 2. Continued

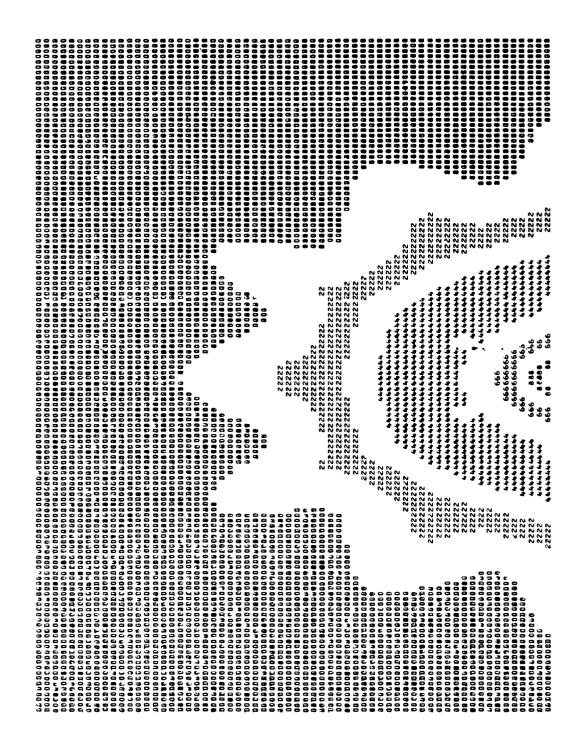


FIGURE 3. R-20, A-10, VIN,VIN2-6, VOUT,VOUT2-5.5, RNIN2-0, RNOUT2-0, W-N/A.

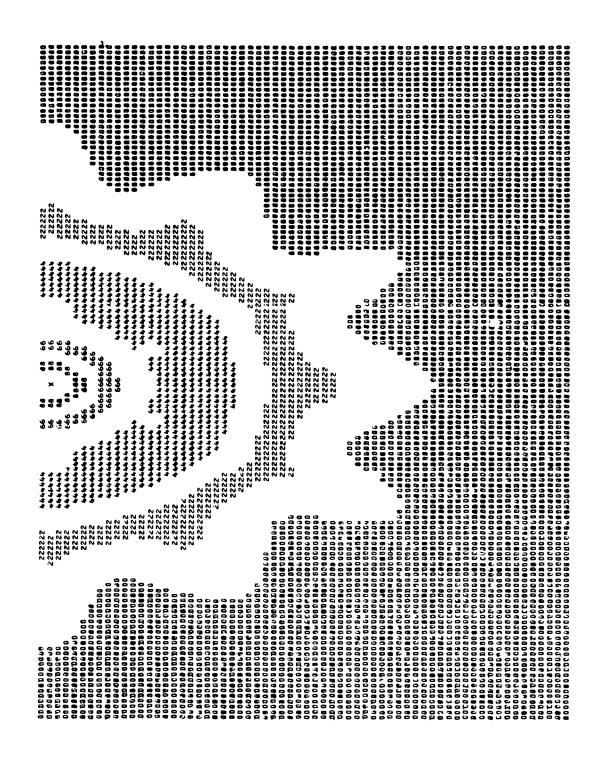


FIGURE 3. Continued

FIGURE 4, R-20, A-10, VIN,VIN2-8, VOUT.VOUT2--2, RNIN2-5, RNOUT2-5, W-N/A.

FIGURE 4. Continued

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FIGURE 5. R-20, A-10, VIN.VIN2-6, VOUT,VOUT2-5.5, RNIN2-5, RNOUT2--5, W-N/A.

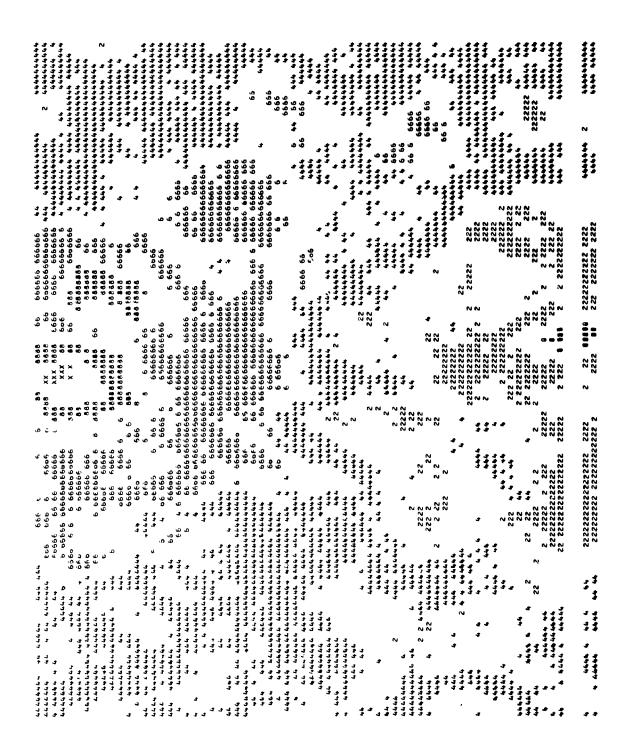


FIGURE 5. Continued

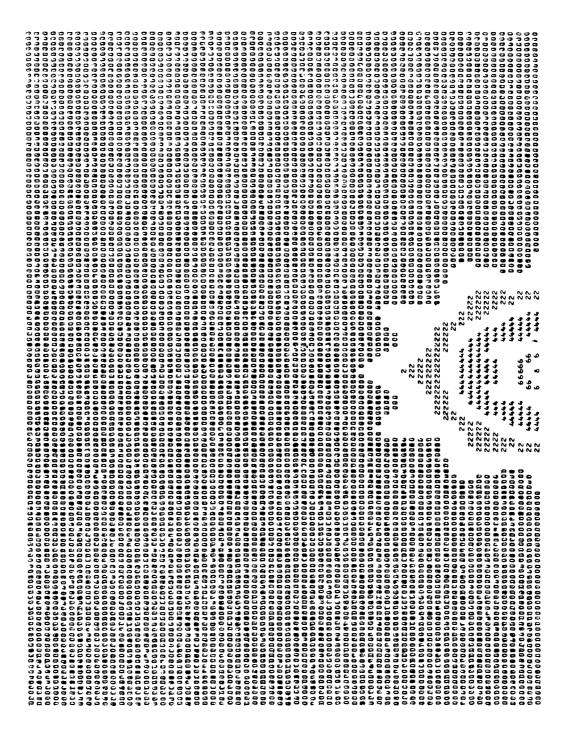


FIGURE 6. R-10, A-5 VIN,VIN2-8, VOUT,VOUT2-2, RNIN2-0, RNOUT2-0, W-N/A.

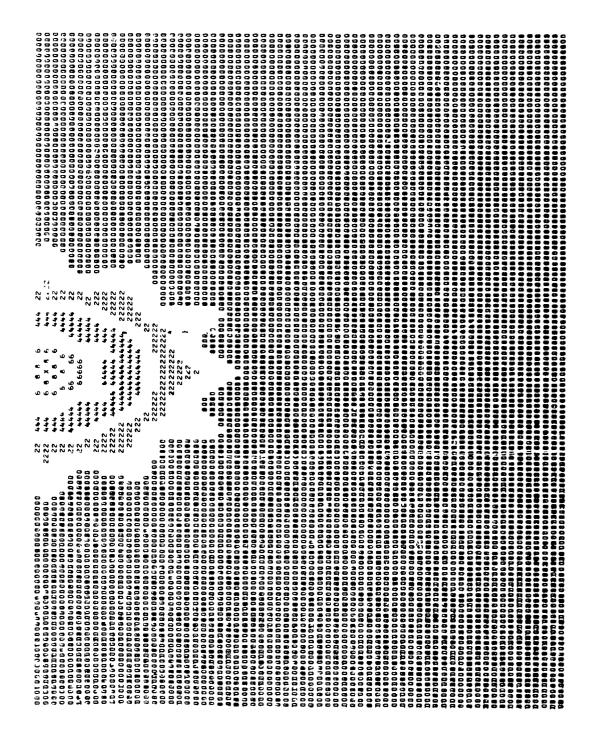


FIGURE 6. Continued

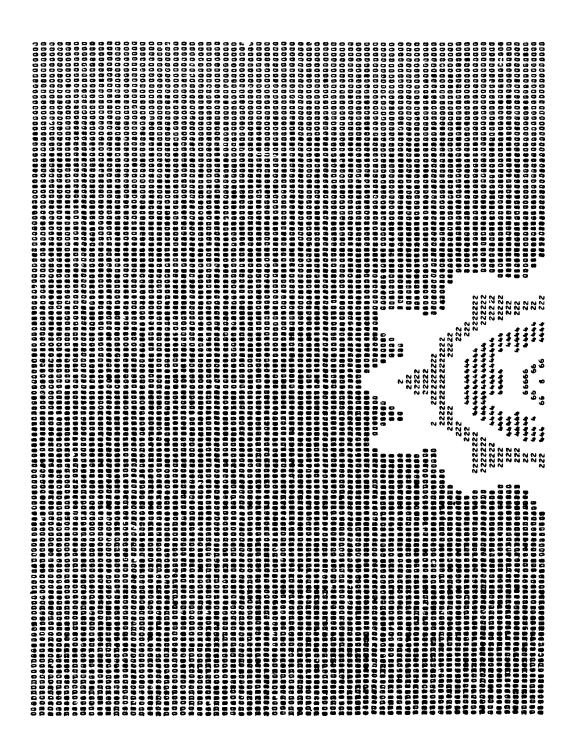


FIGURE 7. R-10, A-5, VIN,VIN2-8, VOUT,VOUT2-2, RNIN2-1, RNOUT2-1, W-N/A.

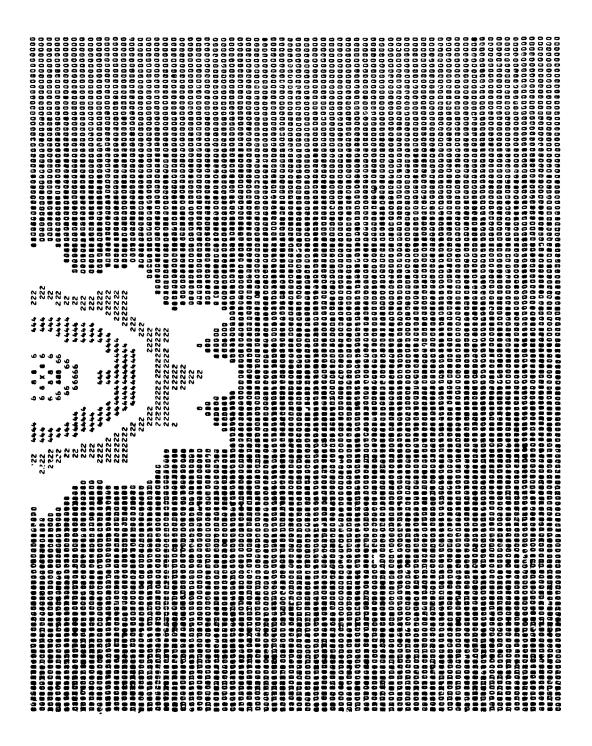


FIGURE 7. Continued



FIGURE 8. R--10, A--5, VIN,VIN2--8, VOUT,VOUT2--2, RNIN2--3, RNOUT2--3, W--N/A.

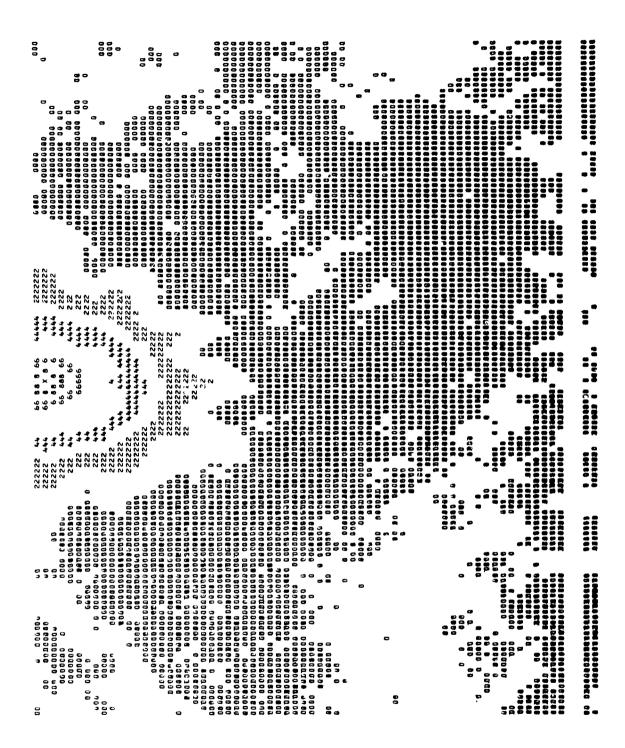


FIGURE 8. Continued

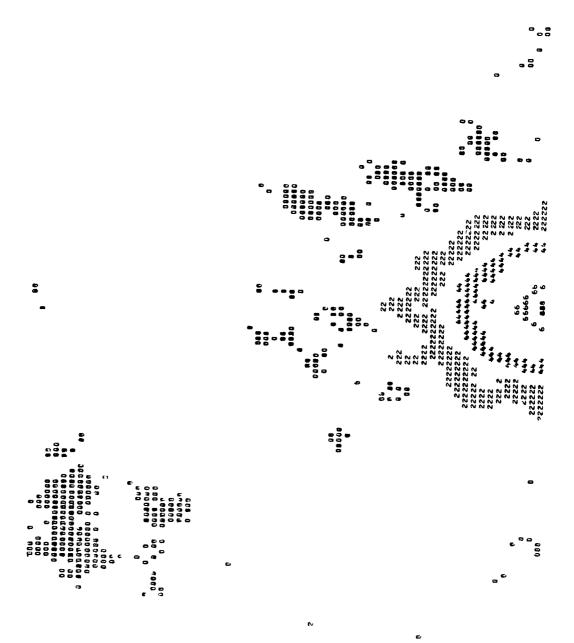


FIGURE 9. R-10, A--5, VIN,VIN2-8, VOUT,VOUT2--2, RNIN2-5, RNOUT2--5, W--N/A.



FIGURE 9. Continued

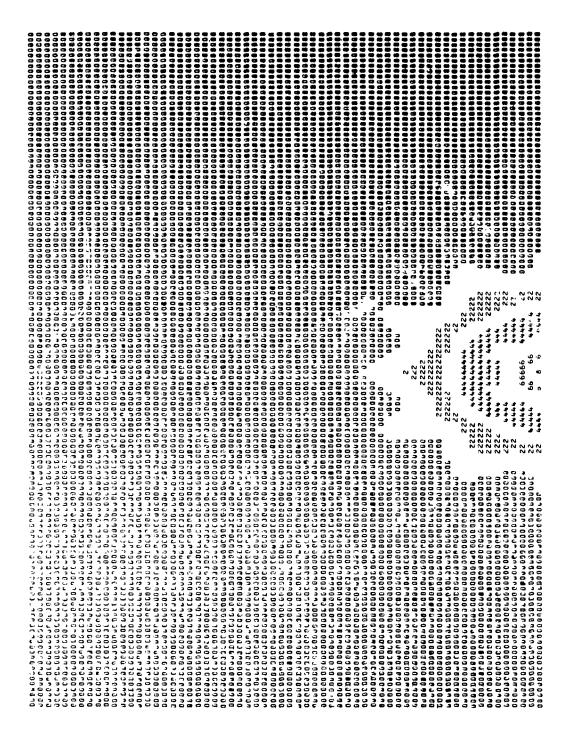


FIGURE 10. R-10, A-5, VIN,VIN2-6, VOUT,VOUT2-5.5, RNIN2-0, RNOUT2-0, W-N/A.

FIGURE 10. Continued

2222222 22222222 222222222 22222222 2222	2222 2222 2222222 22222222 22222222 222222	25.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.	22222222222222222222222222222222222222	222 222 222 222 222 222 222 222 222 22
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FIGURE 11. R-10, A-5, VIN.VIN2-6, VOUT, VOUT2-5.5, RNIN2-1, RNOUT2-1, W-N/A.

FIGURE 11, Continued

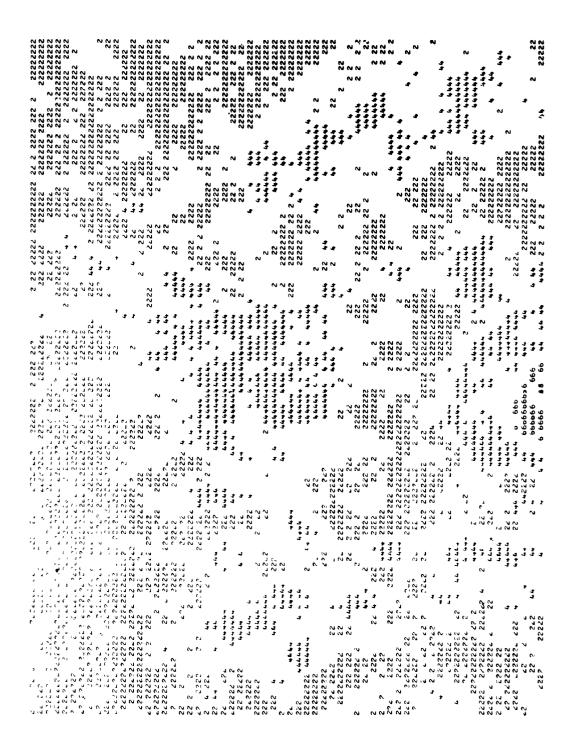


FIGURE 12. R-10, A-5, VIN,VIN2-6, VOUT,VOUT2-5.5, RNIN2-2, RNOUT2-2, W-N/A.

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	2222 2222 2222 2222 2222 2222	2222	22222	2555 2555 2555 2555 2555	22 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2		22222
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	2222	2222	2222	2222	2222	22 22 22 22 22 22 22 22 22 22 22 22 22	22 22
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**	2222	2222	25. 25.				44
2222	2222	7 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	222	70,7	4		12.0
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	22.22	2222	7,77	ייייי אייייי איייייי		444 00 00 00 00 00 00 00 00 00 00 00 00	13
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722 43	4 # 4	22222		22.22	, , , , , , , , , , , , , , , , , , ,	0000000	ر د ک
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FIGURE 12. Continued

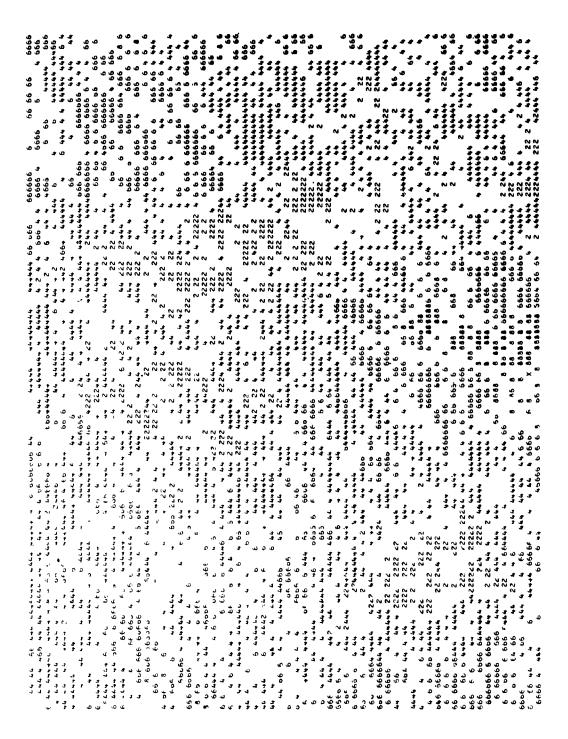


FIGURE 13. R-10, A-5, VIN,VIN2-6, VOUT,VOUΓ2-5.5, RNIN2-5, RNOUT2-5, W-N/A.



FIGURE 13. Continued

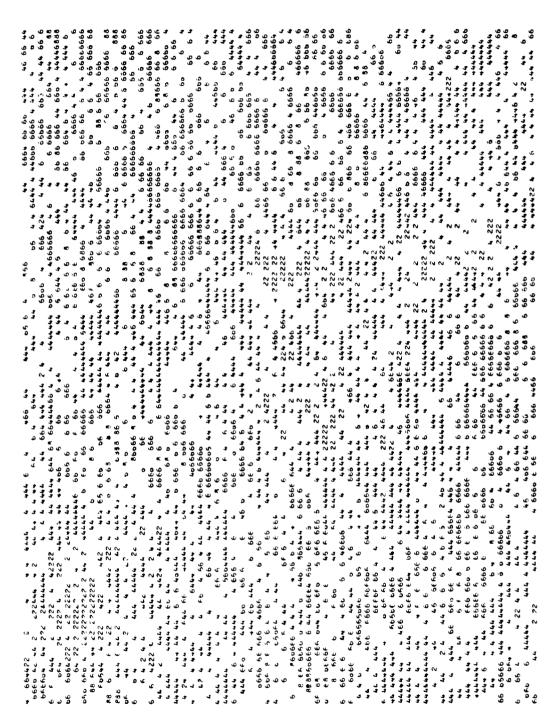
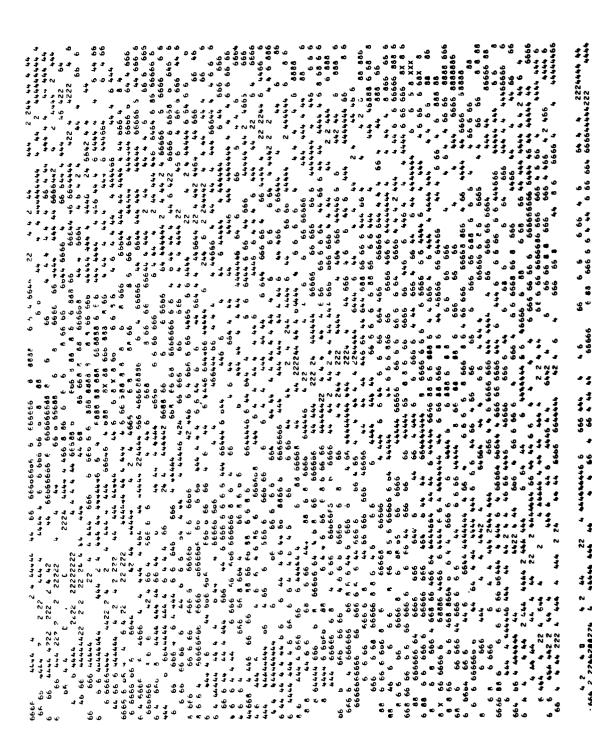


FIGURE 14. R-7, A-3.5, VIN,VIN2-6, VOUT,VOUT2-5.5, RNIN2-5, RNOUT2-5, W-N/A.



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FIGURE 14, Continued

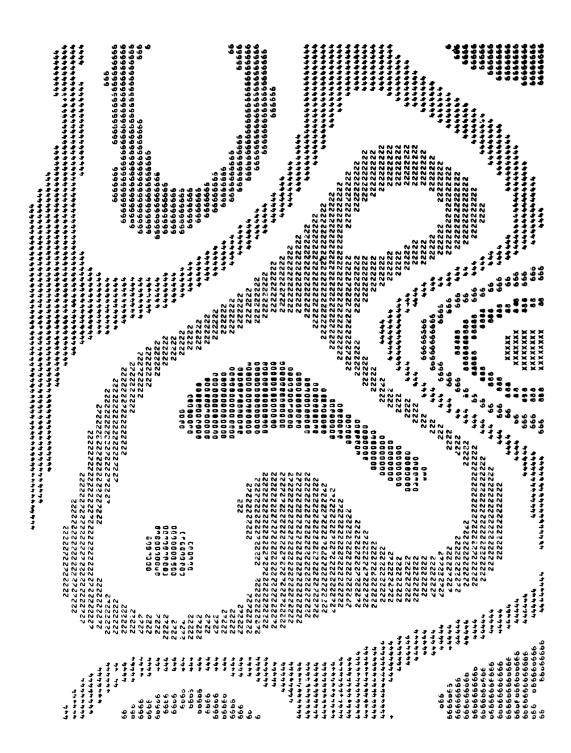
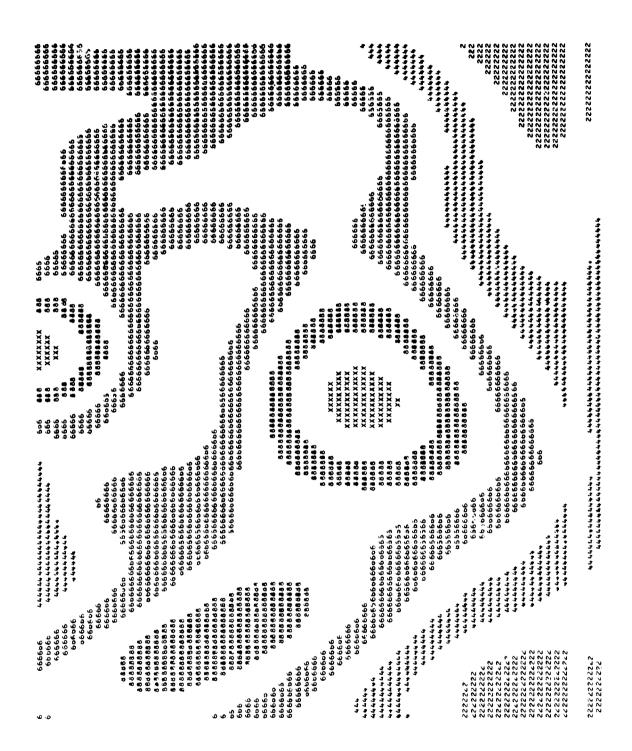


FIGURE 15, R-7, A-3.5, VIN,VIN2-6, VOUT,VOUT2-5.5, RNIN2-5, RNOUT2-5, W-3.



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FIGURE 15. Continued

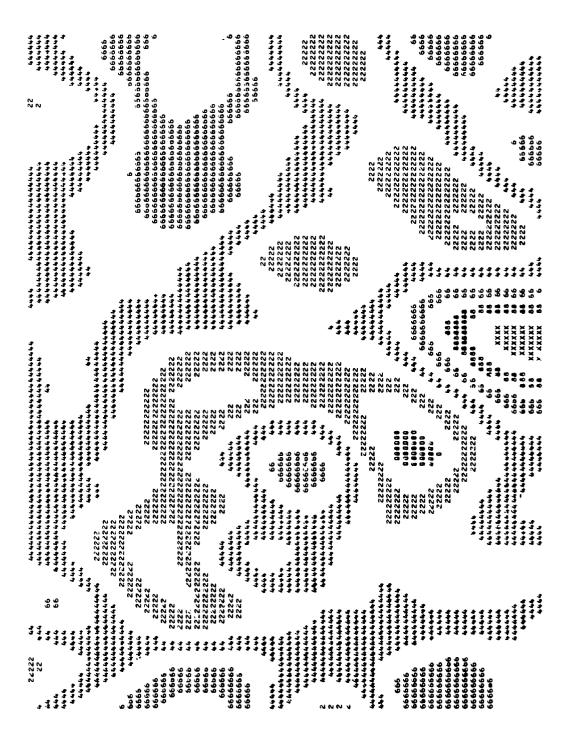


FIGURE 16. R-7, A-3.5, VIN,VIN2-6, VOUT,VOUT2--5.5, RNIN2-5, RNOUT2--5, W-10.

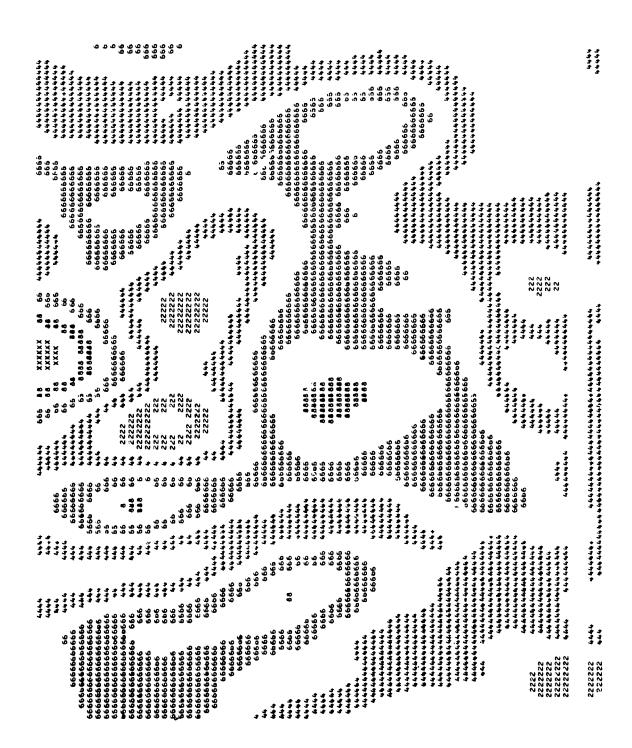


FIGURE 16. Continued

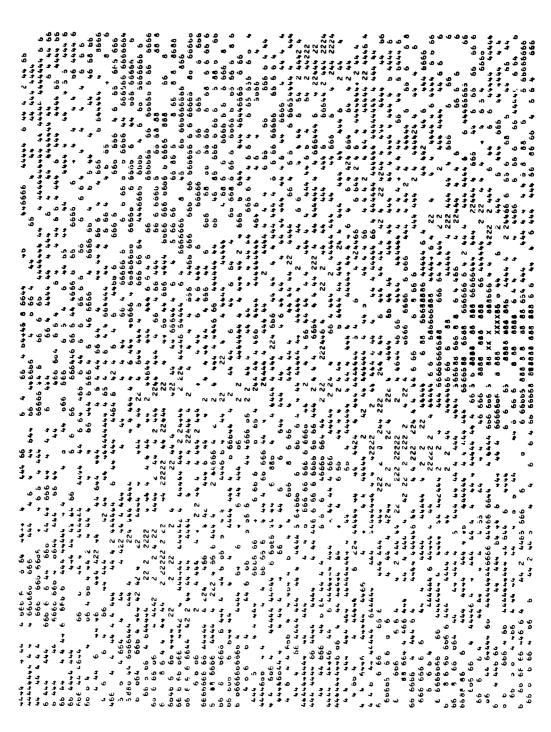


FIGURE 17. R-7, A-3.5, VIN,VIN2-6, VOUT,VOUT2-5.5, RNIN2-5, RNOUT2-5, W-100.

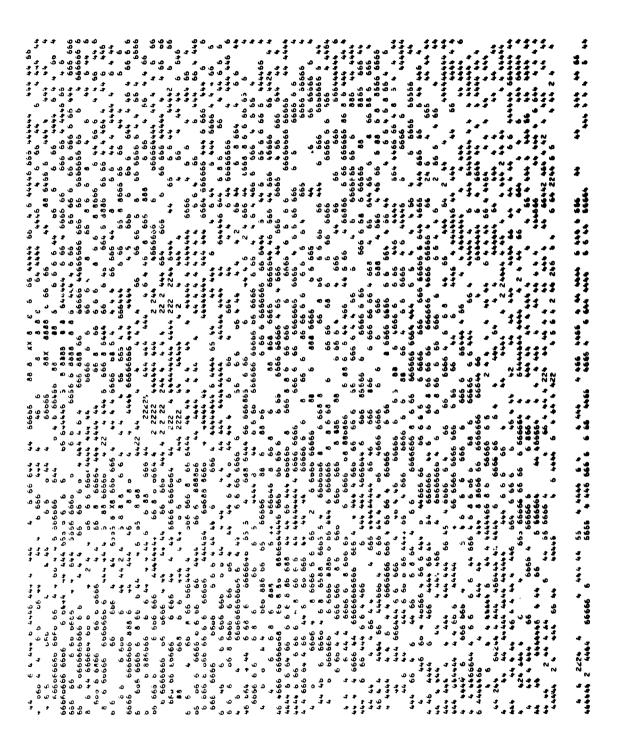
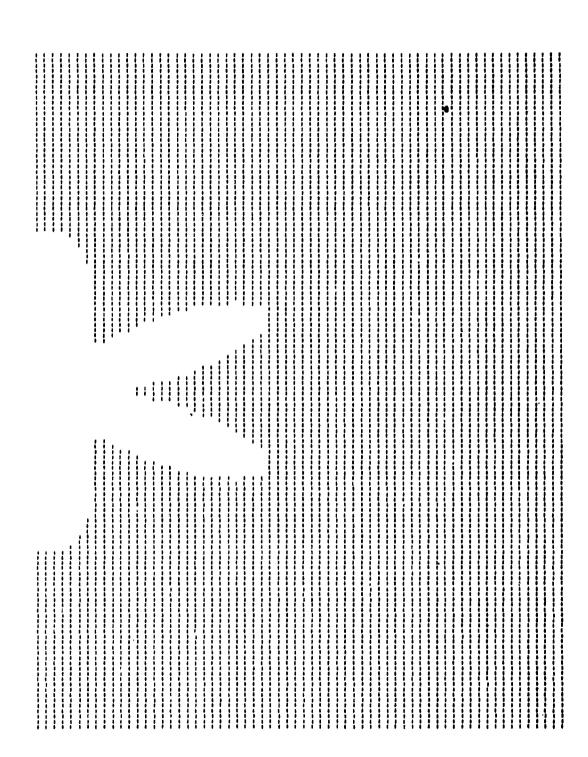


FIGURE 17. Continued

EICURE 10

FIGURE 18.



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FIGURE 18. Continued.

APPENDIX A - LIST OF VARIABLES

EDGE: The length of an edge - in most cases, 128,

R: Radius of the figure; the length from the center of the figure to a

point halfway up one of the arms.

2A: The length of an arm.

NARMS: The number of arms.

(The border of the star is measured from the center of the figure and can be expressed as R + Acos(NARMS·x), where x varies between 0 and 360 degrees, and the distance between the center and the edge of the star thus varies between R - A and R + A).

VIN: Numerical value assigned to points inside the star.

VOUT: Value assigned to points outside the star.

RNIN: Amplitude of random noise assigned inside the star; thus the total

value inside the star will vary between VIN - RNIN and VIN

+ RNIN.

RNOUT: Amplitude of random noise assigned outside the star; thus the

total value outside the star will vary between VOUT - RNOUT

and VOUT + RNOUT.

RRAN: The initial value for the random number generator.

When two figures were being correlated, separate values (VIN2, VOUT2, RNIN2,RNOUT2) were read in. In all our examples, VIN = VIN2 and VOUT = VOUT2, RNIN = RNOUT = 0 (the noiseless image), and RNIN2 and RNOUT2 were varied as

desired.

W: Value used in filtering function; a small value of W produced a

sharp filter (noise effects on correlation substantially reduced), while a large value of W produced a wider, fuzzier filter (noise

effects on correlation only somewhat reduced).

APPENDIX B - LISTING OF COMPUTER PROGRAMS

```
PROGRAF AUTOCOR(INPUT, OUTPUT, TAPES=IMPUT, TAPE6=OUTPUT)

COMPLEX STAR1, STAR2

COMHON STAR1(128,128), STAR2(128,128)

CALL COOL(C)

CALL COOL(C)

CALL COOL(C)

CALL COOL(C)

TREAD 13,N1

13 FORMAT (113)

00 7 I=1,N1

READ 100. EDGE, R, A, NARMS, VIN, VOUT, RNIN, RNOUT, RRAN

ALSO FRINT OUT STARTING VALUES

PRINT 200 **EDGE, R, A, NARMS, VIN, VOUT, RNIN, RNCUT, RRAN

200 FORMAT (/, BH EDGE = *, 1F10.3, 10×, 5H R = *, 1F10.3, /,

PRINT 200 **EDGE, R, A, NARMS, VIN, VOUT, RNIN, RNCUT, RRAN

200 FORMAT (/, BH EDGE = *, 1F10.3, 10×, 5H R = *, 1F10.3, /,

200 FORMAT (/, BH EDGE = *, 1F10.3, 10×, 5H RRAN = *, 1F10.3, /,

20028H VOUT = *, 1F10.3, 10×, 8H RRAN = *, 1F10.3, /,

20039H RNOUT = *, 1F10.3, 10×, 8H RRAN = *, 1F10.3, /,

A FORMAT (4, F10.3, 5X)

PRINT 4, NI, VIN2, VOUT2, RNIN2, RNOUT2

A FORMAT (4, F10.3, 5X)

PRINT 4, NI, VIN2, VOUT2, RNIN2, RNOUT2

4123H VALUE IN BACKGROUND = *, 1F10.3, /, 21H ND1SE IN STARFISH =

CALL STARFSH(EDGE, R, A, NARHS, VIN, VOUT, RNIN, RNOUT, RRAN)

N=1

CALL THODEE(N)
    C
                                                N=1
CALL THODEE(N)
                                               CALL THODEE(4)
NOW HULTIPLY TOGETHER AND TAKE CONJUGATE
CALL MULT
  CC
                                                N=1
                                            CALL THODEE(N)
CALL PUTOUT
CALL POTUUT
CONTINUE
STOP
END
                  SUBROUTINE STAFFSH (EDGE, R, A, NARMS, VIN, VOJT, RNIM. RNOUT, RRAN)

COMPLEX STAR

COMMON STAR(128, 128)

HEDGE=EOGE/128.8

PI=3.14159

TPI=2.0*Pf

DO 1000 II=1, 128

XX=(FLOAT(II)-66.5)*HEDGE

XX=(FLOAT(II)-66.5)*HEDGE

RR=SQR T(XX*XX+YY*Y)

LF(YY.LT.0.0) ALFA=IPI-ALFA

RFISH=RAA*COS(XX/RR)

IF(YY.LT.0.0) ALFA=IPI-ALFA

RFISH=RAA*COS(VARMS*ALFA)

IF(RR-RFISH): 1.2

STAR(II,JJ)=C*PLX(VIN, D.0)

IF(RNIN.NE.0.0)STAR(II,JJ)=CMPLX(VIN+RNIN*(2.0*RANF(RRAN)-1.0),

00 TO 1000

SIAR(II,JJ)=CMPLX(VOUT, D.0)

IF(RNOUT.NE.0.0)STAR(II,JJ)=CMPLX(VOUJ+RNOUT*(2.0*RANF(RRAN)-1.0),

10.0)

CONTINUE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                Cãc4
1000 CONTINUE
                                   KETURN
END
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SUBROUTINE MULT
COMPLEX STAR1, STAR2, GC
COMMON STAP1(128,128), STAR2(128,128)
DO 1 I=1,128
DO 1 J=1,128
CC=STAR1(I,J)*STAR2(I,JL*(1-11**(I+J))
STAR1(I,J)=CONJG(CC)
CONTINUE
RETURN
FNO
              END
              PROGRAM AUTOCORLINPUT, OUTPUT, TAPES=INPUT, TAPE6=OUTPUT)
COMPLEX C(128)
COMPLEX STAR1, STAR2
COMMON STAR1(128, 128), STAR2(128, 128)
    C
               N=1
              CALL THODEE(N)
              N=2
CALL THODES(N)
NOW MULTIPLY TOGETHER AND TAKE CONJUGATE
CALL MULT
NOW FILTER AS OFTEN AS DESIRED
READ 7,N7
DO 300 J=1,N7
CALL STORE
N=2
CALL FILTER(N, EDGE)
CALL OPER(N, EDGE)
CALL POTOUT(N)
CALL POTOUT(N)
CONTINUE
STOP
END
CC
C
     360
               END
```

```
SUBROUTINE FILTER(N, EDGE)
COMPLEX STAR
COMMUN STAR(128, 128, 2)
DIMENSION S(128)
READ 2.W
PORMAT(1510.3)
PRINT 5.W
FORMAT(51 M = .1F10.3)
SIGMA=W*128.0/LDGE
A=128.0/SIGMA
XXP=0.0
IF(A.LE.25.0)XXP=EXP(-A*A)
CONST=1+XXP
CONST=1/CONST
DO 3 I=1,128
A1=(FLOAT(1-1))/SIGMA
YXP=0.0
                                                                            VXP=0.0
IF(A1.LE.25.0)YXP=EXP(-A1*A1)
A2=(FLOAT(129-I))/SIGMA
                                                                      CONTINUE
RETURN
RETURN
DO 4 J=1,128
DO 4 K=1,128
STAR(J,K,N)=STAR(J,K,N)*S(J)*S(K)
CONTINUE
RETURN
ENO
SUBROUTINE OJTPUT
COMPLEX STAR
COMMON STAR(120,120)
DIMENSION ALINE(120), ACHAR(11)
READ 10, (ACHAR(1), I=1,11)

10 FORNAT (AIA)
ENTRY PUTOUT
BIG-RAL(STAR(1,1))
DO 100 JJ=1,120
DO 100 JJ=1,120
DO 100 JJ=1,120
DO 100 JJ=1,120
If (2.C, SMALL) SMALL=Z

100 CONTINUE
RAMC=BIG-SHALL
PRANLISHALL SMALL=Z

110 FORNAT 10 H1)
DO 200 JJ=1,120
H=10,00 HEAL(STAR(II,JJ))-SMALL)/RANGE+L.5
ALINE(II)=RCHAR(NN)

150 CONTINUE
PRINT 160, (ALINE(I), I=1,120)
200 CONTINUE
PRINT 150, SMALL, BIG, RANGE
250 FORNAT (/////, 18H SMALLEST WALUE
ENTRY POTUUT
PRINT 270
271 FORNAT (/////, 18H SMALLEST WALUE
ENTRY POTUUT
PRINT 270
272 FORNAT (AIA)
RETURN
ENTRY POTUUT
PRINT 270
274 FORNAT (SALL)
300 FORN
```

SUBROUTINE SWITCH
COMPLEX STAR1, STAR2
COMMON STAR1(128,128), STAR2(128,128)
DO 700 JI=1,128
DO 700 JJ=1,128
STAR2(II, JJJ=STAR1(129-II,129-JJ1
Tun CONTINUE
RETURN
END

THE STATE OF THE S

SUBROUTINE TAGDEE(N)

LOMPLEX C(128)

LOMPLEX STAR

COMMON STAR(128,128,2)

DO 200 I=1,128

OO 100 J=1,128

C(J)=STAR(J,I,N)

100 CONTINUE

NOW THE COLUMN WILL BE TRANSFORMED

CALL TRANS(C)

NOW THE IRANSFORMED COLUMN WILL BE PUF BACK IN THE ARRAY

DO 200 JJ=1,128

STAR(JJ,I,N)=C(JJ)

200 CONTINUE

NOM EACH ROW OF THE NEW ARRAY WILL BE TRANSFORMED

DO 400 L=1,128

C(K)=STAR(L,K,N)

300 CONTINUE

CALL TRANS(C)

DO 400 M=1,128

STAR(L,H,N)=C(H)

400 CONTINUE

CALL TRANS(C)

CONTINUE

CALL TRANS(C)